EEN 360
MODERN
CONTROL

SPRING 2008
TEST 1
50 MINUTES

NDU
ECCE DEPARTMENT
S.C ELMURR

15/100

NAME:

NOTE 1: OPEN BOOK, OPEN NOTES, CLOSED OLD TESTS AND SOLUTIONS.
NOTE 2: SHOW ALL WORK IN ORDER TO GET FULL CREDIT.

1) 17
2) 30
3) 18

1. 20 pts: Find the inverse Laplace transform of the following function. Perform partial fraction expansion.

$$G(s) = \frac{10}{(s+1)^2(s+3)}$$

$$G(s) = \frac{A_1}{(s+3)} + \frac{A_2}{(s+1)^2} + \frac{A_3}{(s+1)}$$

$$A_1 = \left| G(s)(s+3) \right|_{s=-3} = \frac{10}{(-2)^2} = 2.5$$

$$A_2 = \left| G(s)(s+1)^2 \right|_{s=-1} = \frac{10}{(2)} = 2.5$$

$$A_3 = \frac{d}{ds}\left[\frac{10}{s+3}\right]\Bigg|_{s=-1} = \frac{-10}{(s+3)^2}\Bigg|_{s=-1} = -\frac{10}{(2)^2} = -2.5$$

$$G(s) = \frac{2.5}{(s+3)} + \frac{2.5}{(s+1)^2} - \frac{2.5}{(s+1)}$$

-3

$$G(t) = \left(2.5e^{-3t} + 2.5e^{-2t} - 2.5e^{-t}\right)u(t) \quad t \geq 0$$

$5t\,e^{-t}$

2. 35 pts. Apply the gain formula to the SFG shown in fig. P3 to find the following transfer Functions:

Y_7/Y_1, Y_7/Y_2

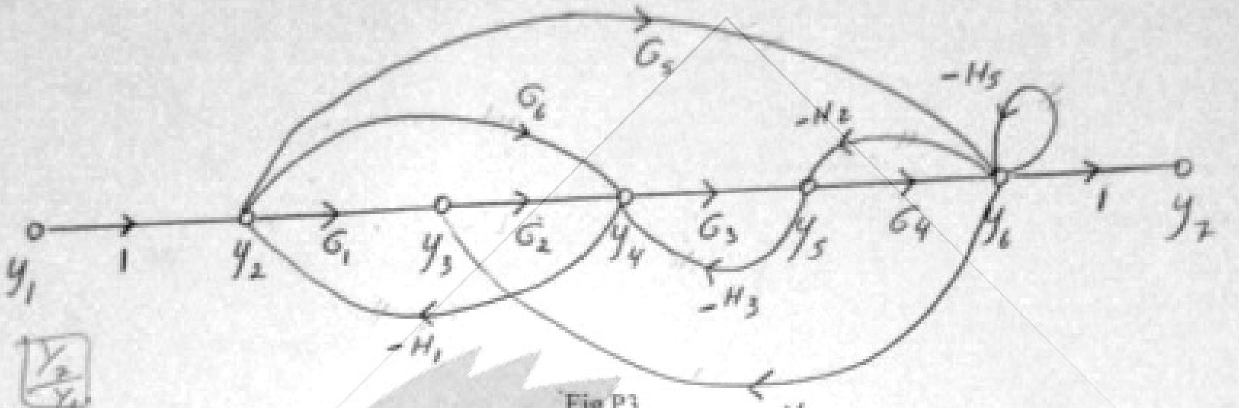


Fig.P3.

$\boxed{\frac{Y_7}{Y_1}}$

Path:

$M_1 = G_1 G_2 G_3 G_4$

$M_2 = G_6 G_3 G_4$

$M_3 = G_5$

Δm:

$\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1 - [-H_3 G_3] = 1 + H_3 G_3$

Loops:

$L_1 = -G_1 G_2 H_1$

$L_2 = -G_2 G_3 G_4 H_4$

$L_3 = -G_3 H_3$

$L_4 = -G_4 H_2$

$L_5 = -H_5$

$L_6 = -G_6 H_1$

$L_7 = -G_5 H_2 H_3 H_1$

Δ:

$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + \ldots$
$+ [H_5 G_3 H_3] + [H_5 G_6 H_1]$
$+ [H_5 G_1 G_2 H_1] + [G_6 H_1 H_2 G_4]$
$+ [G_1 G_2 H_1 G_4 H_2]$

$$\frac{Y_7}{Y_1} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta}$$

$$\frac{Y_7}{Y_2} = \frac{Y_7}{Y_1}\,\frac{Y_1}{Y_2}$$

-5

$\boxed{\frac{Y_2}{Y_1}}$ Path: $M_1 = 1$

Loop: Same

Δm: $\Delta_1 = 1 - [-H_3 G_3 - H_2 G_4 - H_5 - H_4 G_2 G_3 G_4] + G_3 H_3 H_5$

Δ: Same

$$\frac{Y_2}{Y_1} = \frac{M_1 \Delta_1}{\Delta}$$ ok!

$$\frac{Y_7}{Y_2} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{M_1\Delta_1}$$

3. 45 pts. For the transfer function shown below, determine the value of K so that the Bode magnitude plot intersects at -8 dB at w = 0.1 rad/s. Plot the magnitude and phase Bode approximations. Find the 0-db crossings and label the gain at each break frequency. Find the gain and phase margins.

$$GH(s) = \frac{Ks(s+2)^2(s+100)(s+800)^2(s+5000)}{s(s+10)^2(s+40)(s+500)^2(s+1000)(s+20000)^2}$$

$$GH(s) = \frac{K\,1.28\times10^{12}\,s\left(1+\frac{s}{2}\right)^2\left(1+\frac{s}{100}\right)\left(1+\frac{s}{800}\right)^2\left(1+\frac{s}{5000}\right)}{4\times10^{20}\left(1+\frac{s}{10}\right)^2\left(1+\frac{s}{40}\right)\left(1+\frac{s}{500}\right)^2\left(1+\frac{s}{1000}\right)\left(1+\frac{s}{20000}\right)}$$

$20\log(K\ 0.32\times10^{-8}) = \cancel{-8}$

$K(0.32\times10^{-8}) = 10^{-0.4}\ \log$

$K = 1.24\times10^{-8}$

$20\log(K\ 0.32\times10^{-4}) = 12$ → Btitkhand bal 0.01 rad/s

$K = 124408490.8$

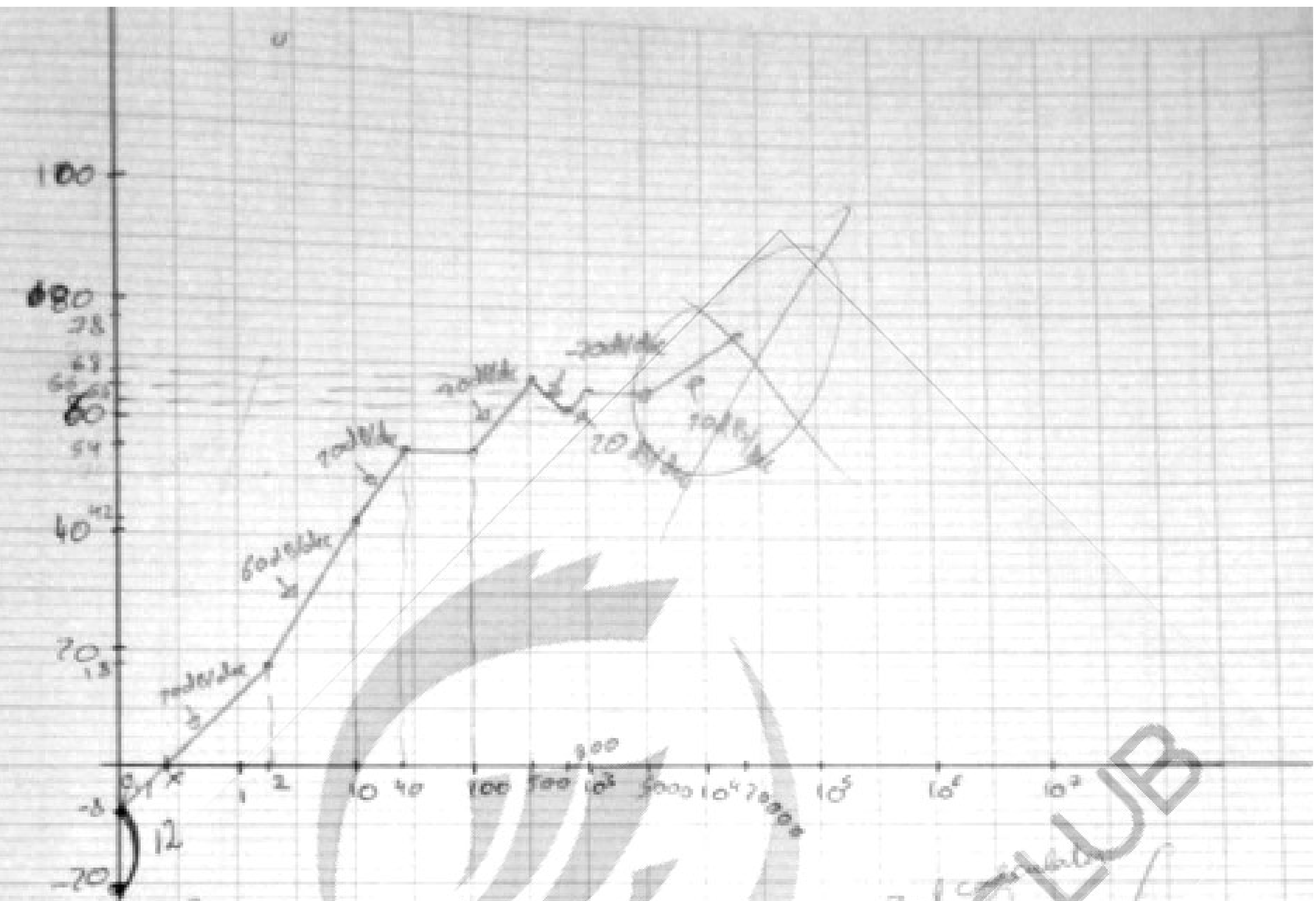
$\frac{1.28\times10^{12}}{4\times10^{20}} = 3.2\times10^{-9}$

$20\log(K\ 3.2\times10^{-9}) =$

-8

-20

$20\log K = -8$



$60(\log 2 - \log 10) + 18 = Y$ at 10

$\cancel{60(\log 10 - \log 2) = 42}$

$\boxed{20(\log 2 - \log 0.1) - 8 = 18}$

$(0.1, -8)$

$K = 20\log$ $(x, 0)$

$8 = 20\{\log x - \log$

$\log x = 0.4 - 1$

$= -0.6$

$x = 0.25$

-40

-60

-80

-100

$20\log(3.2 \times 10^{-9} K) = 12$

Final slope :

at -20 dB